Network Effects and Multi-Network Firms' Dynamic Pricing

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Abstract

This paper studies multi-network firms' dynamic pricing strategy when products are subject to network effects. Compared with single-network firms, multi-network firms prefer network concentration, have greater market power, and face spillover effects across firms. We setup a finite-horizon two-network model to compare the pricing strategies of single- and multi-network firms. We find that multi-network firms set lower prices for larger networks than for the smaller networks. This pricing strategy is the opposite of single-network firms' strategy. As a result, the network market is more concentrated with multi-network firms than with single-network firms. Using the smartphone price data for U.S. from 2011 to 2013, we show that the multi-network telecom carriers choose lower prices for the smartphones with larger operating system networks.

Keywords: network effect, multi-network firms, dynamic pricing, network concentration.

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1 Introduction

In many industries, goods are subject to network effects, indicating a positive externality among the users of a good. Two types of firms exist in these markets, single- and multi-network firms. A single-network firm sells goods that are all associated with the same network. Multi-network firms sell goods that are associated with different networks. The literature on network effects has focused on single-network firms and paid little attention to multi-network firms. However, multi-network firms exist in many markets. For example, smartphones are subject to operating system (OS) network effects (e.g., iOS and Android), and telecommunication carriers sell multiple OSs' models (e.g., AT&T and Verizon). Television networks (e.g., ESPN and NBC) generate network effects through channels, and cable and satellite companies offer multiple television networks to subscribers. Network effects also exist for automobile manufacturers, and a dealer can sell multiple manufacturers' models. To fill the gap, we study the multi-network firms' dynamic pricing strategy in this paper.

The two types of firms face both similar and different factors when choosing prices. Katz and Shapiro (1986) and Klemperer (1987) study single-network firms' pricing equilibrium. They point out that single-network firms face a trade-off between investing in future network size with low prices and harvesting the current network size with high prices. This trade-off also applies to multi-network firms. A multi-network firm faces this trade-off for multiple networks.

However, multi-network firms' pricing problem is more complex in three aspects. First, a multi-network firm's profit can increase with network concentration. As concentration increases, consumers' willingness to pay for the larger network rises, which increases firms' profits. To achieve greater concentration, multi-network firms have an incentive to choose relatively lower prices for the large networks than for the small networks. This strategy is the opposite of single-network firms' pricing strategy. Cabral

(2011) finds that the greater market power of large single-network firms allows them to choose higher prices than small single-network firms.

Second, multi-network firms have greater market power and can internalize the competition across networks. When a single-network firm increases the price of its network, all switching consumers will choose other networks or the outside option. However, when a multi-network firm increases the price of a network, some switching consumers will choose the firm's other networks. This gives the multi-network firm greater market power than single-network firms, and they set higher prices for all networks, compared with single-network firms.

Third, when multi-network firms compete on the same networks, one firm's low prices increase the network sizes, which also benefit other multi-network firms. Thus, a positive spillover effect exists among multi-network firms. This weakens their incentives to invest in network size by choosing low prices,. Therefore, multi-network firms' pricing problems are more complex.

To study multi-network firms' pricing strategies, we compare single- and multi-network firms' dynamic pricing strategies in a finite-horizon model. On the demand side, consumers make discrete choices among two networks and an outside option. Once a consumer purchases a product, he exits the market in the following periods. Thus, the market size evolves with the network sizes. On the supply side, we consider two settings, with single-network firms and multi-network firms, respectively. In the first setting, we consider two single-network firms, each of which sells a good that is associated with a different network. In each period, the firms choose the goods' prices, and the two firms play a dynamic pricing game. In the second setting, we consider a monopolist multi-network firm that chooses the prices for the two goods in each period. We analyze the impacts of competition on multi-networks' prices using numerical examples.

The main findings are as follows. First, multi-network firms choose a lower price for the initially larger network than for the smaller network before the last period. This counter-intuitive strategy is due to the impact of multi-network firms' preference for network concentration. Multi-network firms can obtain greater long-run profits as the network concentration increases. Second, with single-network firms, the initially larger network has a higher price than the smaller network in each period. This is similar to the finding in Cabral (2011), although the demand model is different in this paper. In addition, we find that, although the larger network has a higher price, it can keep the network size advantage throughout the game. Third, the network market is more concentrated in the multi-network firm case, compared with the single-network firm case.

The multi-network firms' pricing strategy shows up in the smartphone industry. Smartphones are subject to network effects at the OS level (Bresnahan, Orsini, and Yin (2014), Sinkinson?, Luo (2022)). The telecom carriers act like multi-network firms in the U.S. since they each sell multiple OSs' smartphone models. Using monthly smartphone price data from 2011 to 2013, we find that the carriers' prices for the larger OSs (iOS and Android) are lower than the prices for the smaller OSs (Blackberry and Windows). This finding is consistent with the multi-network firms' pricing strategy in the theoretical model.

This paper is closely related to Cabral (2011), which studies two single-network firms' pricing game in an infinite-horizon game. Cabral (2011) assumes that the market size is constant across periods, and consumers do not have an outside option. The model in this paper is different in three ways. First, the market size evolves endogenously in this paper. As more consumers join the networks, the amount of prospective consumers decreases. This implies that the firms' prices will affect the future market size and thus the long-run total profits. Second, consumers have an outside option in each period, which is to not join any networks. Third, this paper considers a finite-horizon game due to the non-stationarity of the firms' profit-maximization problems. As more and more consumers join the networks, the market size decreases over time. Thus, we consider a finite-horizon game in this paper.

This article contributes to the network effect literature by studying the multi-network firms' dynamic pricing strategy. Existing theoretical research on network effects has focused on single-network firms. Many papers studied the pricing strategies of monopolistic and oligopolistic single-network firms, including Katz and Shapiro (1985), Farrell and Saloner (1986), Katz and Shapiro (1992), Katz and Shapiro (1994), Shapiro and Varian (1999), Rochet and Tirole (2003), Armstrong (2006), Rochet and Tirole (2006) Zhu and Iansiti (2007), Rysman (2009), Weyl (2010), and Cabral (2011). This paper is the first to analyze multi-network firms' pricing strategy. We find that multi-network firms' dynamic pricing strategy is the opposite of the single-network firms' strategy.

The empirical literature has studied markets where single-network manufacturers choose retail prices, including Rysman (2004), Park (2004), Nair, Chintagunta, and Dubé (2004), Ackerberg and Gowrisankaran (2006), Dubé, Hitsch, and Chintagunta (2010), Lee (2013), and Gowrisankaran, Park, and Rysman (2014). In these studies, firms cannot cannot choose prices for multiple networks or set differentiated prices across networks. In this paper, we focus on the multi-network firms, for whom the ability to internalize competition across networks and set differentiated prices plays an important role.

The paper proceeds as follows. Section 2 describes a discrete choice demand model of consumers choosing the networks. In section 3, we set up the two supply settings in a finite-horizon framework and compare the pricing strategies in the two settings. We also compare the degree of network concentration between the two settings. In section 4, we increase the number of periods in the dynamic game and use numerical examples to show the two types of firms' pricing strategies. In section 5, we consider competition among multi-network firms. In Section 6, we use smartphone price data to analyze the multi-network telecom carriers' pricing strategies. Section 7 concludes the paper.

2 Consumer Demand

Consider two durable goods with network effects, A and B. They are associated with two different networks. To simplify notation, we denote their networks by $\{A, B\}$. Consumers can purchase the goods in any period $t \in \{1, 2, ..., T\}$. Once a consumer purchases a good, he joins the associated network and does not enter the market again. Let the total mass of consumers be one. Denote the two networks' market shares at the beginning of period t by $\boldsymbol{n}_t = (n_{At}, n_{Bt})$. Assume that, some consumers are not on either of the two networks at the beginning of the first period, $n_{A1} + n_{B1} < 1$. In each period, only the consumers who have not joined any network enter the market and considers whether to purchase a good. Thus, the market size in period t is $M_t = 1 - n_{At} - n_{Bt}$.

Consumer i's utility from purchasing good $j \in \{A, B\}$ in period t is

$$u_{ijt} = \delta_j + \gamma n_{jt} - \alpha p_{jt} + \epsilon_{ijt},$$

where δ_j measures good j's quality, p_{jt} is the price of good j, and ϵ_{ijt} is an idiosyncratic utility shock. The parameter, $\gamma (> 0)$, measures the network strength, and γn_{jt} is the utility from the network effect of good j. In addition to the two goods, an outside option exists, which means not buying either of the two goods. Consumer i's utility of the outside option is $u_{i0t} = \epsilon_{i0t}$. The mean utility of the outside option is zero, and ϵ_{i0t} is the utility shock. To focus on the network effect, we assume that the two goods have the same quality, $\delta_A = \delta_B$.

Assume that ϵ_{ijt} follows the Type-I extreme value distribution and is independent and identically distributed (i.i.d.) across consumers, goods, and time periods. Given these assumptions, the sales market share of good $j \in \{A, B\}$ in period t is

$$s_{jt}(p_{At}, p_{Bt}; \boldsymbol{n}_t) = \frac{\mathrm{e}^{(\delta_j + \gamma n_{jt} - \alpha p_{jt})}}{1 + \sum_{k=A,B} \mathrm{e}^{(\delta_k + \gamma n_{kt} - \alpha p_{kt})}}. \tag{1}$$

This share increases with δ_j and n_{jt} and decreases with p_{jt}. The two networks grow due to new sales of the goods. Due to the durability of the goods, consumers exit the market once they purchase the goods. At the beginning of the period $t+1$, each firm's network size is the sum of the network size in period t and the new consumers in period t. The market share of network j is

$$n_{jt+1} = n_{jt} + M_t s_{jt}(p_{At}, p_{Bt}; \boldsymbol{n}_t). \tag{2}$$

The literature on network effects often uses other demand models to analyze firms' pricing strategies. Many studies use the Hotelling framework to model consumers' demand for heterogeneous products. This paper does not apply the Hotelling model for two reasons. First, the Hotelling model does not consider the outside option for consumers. However, allowing for the outside option is important in the dynamic evolution of the network sizes and market size in this paper. It is different to derive the equilibrium demand when introducing the outside option to the Hotelling model. The discrete choice model is more tractable when considering the outside option. Second, extending the static Hotelling model to a dynamic framework makes it very challenging to solve for the subgame perfect Nash equilibrium of the firms' dynamic game.

Cabral (2011) uses a stylized demand model to analyze single-network firms' dynamic pricing game. He makes two assumptions. First, in each period, a new consumer enters the market and an existing consumer dies. This assumption does not allow for variation in the market size over time. In this paper, the market size evolves with the two networks' sizes. As more consumers adopt the goods, the market size decreases over time. Second, the new consumer does not have an outside option. This assumption diverges from the reality in which consumers may choose to not join any network. The discrete choice model allows the market size to evolve over time and considers the outside option.

3 Single- and Multi-Network Firms' Dynamic Pricing

In this section, we analyze two supply settings in a finite-horizon two-network model, with single- and multi-network firms, respectively. The goal is to compare the pricing strategies of the two single-network firms. We first show that a multi-network firm sets a lower price for the smartphone with the larger network and single-network firms choose the opposite pricing strategy. We then show that the network market becomes more concentrated in multi-network firm setting than in the single-network firm setting.

3.1 Single-Network Firms

Consider two single-network firms, A and B. Firm A sells good A, and firm B sells good B. Assume that the firms have constant marginal costs, (c_A, c_B). They play a dynamic pricing game. Firm $j \in \{A, B\}$'s profit in period t is

$$\pi^s_{jt}(p^s_{At}, p^s_{Bt}; \boldsymbol{n}_t) = (p^s_{jt} - c_j)s_{jt}(p^s_{At}, p^s_{Bt}; \boldsymbol{n}_t)M_t,$$

where the superscript s denotes the single-network firm case. In period $t(< T)$, the profit maximization problem of firm j is

$$\underset{p^s_{jt}}{max}\{\pi^s_{jt}(p^s_{At}, p^s_{Bt}; \boldsymbol{n}_t) + \beta^d V^s_{jt+1}(\boldsymbol{n}_{t+1}; \boldsymbol{n}_t, \boldsymbol{p}^s_t)\},$$

where $\boldsymbol{p}^s_t = (p^s_{At}, p^s_{Bt})$ is the price vector, and β^d is the discount factor. The network size evolves according to the transition rule in equation (2). The future value function, V^s_{jt+1}, is firm- and time-specific. It is firm-specific because the firms have asymmetric network sizes. It is time-specific because the number of periods left varies over time in the finite-horizon game.

The first-order condition (FOC) with respect to p^s_{jt} is

$$s_{jt}M_t - (p^s_{jt} - c_j)\alpha s_{jt}(1 - s_{jt})M_t + \beta^d \frac{\partial V^s_{jt+1}}{\partial p^s_{jt}} = 0, \tag{3}$$

where $-\alpha s_{jt}(1-s_{jt})$ is the partial derivative of s_{jt} with respect to (w.r.t.) p^s_{jt}. The first term in the FOC is the markup effect on the current profits. When p^s_{jt} increases, firm j's markup on the good increases by the same amount. The second term is the impact of price on the current sales. As p^s_{jt} increases, the demand (s_{jt}) decreases, leading to a negative impact on the current profits. The last term in the FOC is the impact of p^s_{jt} on firm j's future profits.

In period $t < T$, the prices in period t affect V^s_{jt+1} through n_{At+1} and n_{Bt+1}.[1] For example, increasing p_{At} has a few impacts on the firm's future profits. First, a higher p_{At} reduces n_{At+1} due to lower s_{At} in period t, which has a negative impact on the consumers' utility from buying A and firm A's profits in period $t+1$. Second, a higher p_{At} increases the sales of good B and n_{Bt+1}. This has a negative impact on the network size and firm A's profits in period $t+1$. Third, a lower n_{At+1} implies that more

[1]That is, $\frac{\partial V^s_{jt+1}}{\partial p^s_{jt}} = \frac{\partial V^s_{jt+1}}{\partial n_{At+1}} \frac{\partial n_{At+1}}{\partial p^s_{jt}} + \frac{\partial V^s_{jt+1}}{\partial n_{Bt+1}} \frac{\partial n_{Bt+1}}{\partial p^s_{jt}}$.

consumers enter the market in period $t+1$, which has a positive impact on firm A's profits in period $t+1$.

At the same time, the two firms compete with each other. A lower p_{At} helps firm A to attract more consumers and reduces firm B's sales. This increases firm A's network size in period $t+1$. Both firms have this incentive to use low prices to invest on the future network sizes. In equilibrium, the initial network advantage of firm A gives it greater market power than firm B since consumers obtain higher utility from good A when the network effect exists ($\gamma > 0$). This leads to a higher equilibrium price for firm A than firm B in each period.

In period T, both firms only maximize the current period's profits. Thus, firm j's profit maximization problem is $\underset{p_{jT}^s}{max}\{\pi_{jT}^s(p_{AT}^s, p_{BT}^s; \boldsymbol{n}_T)\}$. The FOC w.r.t. p_{jT}^s is

$$s_{jT}M_T - (p_{jT}^s - c_j)\alpha s_{jT}(1 - s_{jT})M_T = 0. \tag{4}$$

The firm's future value is 0 in period T. When choosing the optimal price, the firms only face the trade-off between the markup effect and the sales effect. As in equation (3), the first term is the impact of the markup effect on profits, and the second term is the impact of the sales effect on profits.

Without loss of generality, we consider the case in which A has a higher market share than B in the first period, $n_{A1} > n_{B1}$. This initial asymmetry in network size can arise if the networks enter the market at different time periods or if they have experienced different demand or cost shocks. We maintain this assumption in all following sections. We use backward induction to solve for the equilibrium prices. Denote the single-network firms' equilibrium prices in period t by $\{p_{At}^{*s}, p_{Bt}^{*s}\}$. In period T, we use the FOCs to solve for the prices explicitly. The equilibrium prices, $\{p_{AT}^{*s}, p_{BT}^{*s}\}$, are functions of (n_{AT}, n_{BT}). In period $t < T$, solving for the prices involves $V_{jt+1}^s(n_{At+1}, n_{Bt+1})$. We obtain $V_{jt+1}^s(n_{At+1}, n_{Bt+1})$ by solving for the equilibrium prices in all following periods and calculating the total discounted profits.[2]

To prove this result analytically, we consider a two-period game ($T = 2$) in the rest of this section. Due to the dynamic impacts of current prices on the future network sizes, it is very challenging to solve for the equilibrium prices when $T \geq 3$ analytically. Thus, we first study the two types of firms' equilibrium pricing strategies with $T = 2$ in this section. In the next section, we extend the model to cases with $T \geq 3$ and analyze the equilibrium pricing strategies numerically.

Proposition 1. *(1) In the subgame perfect Nash equilibrium, the price of A is higher than the price of B in both periods. That is, $p^{*s}_{At} > p^{*s}_{Bt}$, for $t = 1, 2$.*
(2)Firm A keeps its network advantage in the second period, $n_{A2} > n_{B2}$.

Proof. See section A.1 of the Appendix for the proof. □

Firm A chooses higher prices because it has the initial OS network size advantage. Suppose that $p_{At} = p^{s*}_{Bt}$, then firm A's marginal profits from increasing p_{At} is positive due to higher demand for network A. In this case, firm A can increase profits by increasing its price. While this finding is similar to the results in **?**, the model assumptions in this paper are very different. Although firm A chooses a higher price than firm B in the first period, it can keep the network advantage in the second period. Proposition 1 holds as long as the discount factor β^d is positive.

3.2 A Monopoly Multi-Network firm

Now consider a multi-network firm who sells the two networks' products and chooses prices of them to maximize the long-run total profits. The firm's profit in period t is

[2]For $t = T - 1$, we can derive the expression for $V^s_{jt+1}(n_{At+1}, n_{Bt+1})$ analytically by solving for the prices in period T. For $t < T - 1$, deriving the expression for $V^s_{jt+1}(n_{At+1}, n_{Bt+1})$ becomes very difficult. Therefore, we analyze the game when $T = 2$ analytically and use numerical examples when $T \geq 3$.

the sum of the profits from the two products:

$$\pi_t(p^m_{At}, p^m_{Bt}; \boldsymbol{n}_t) = [(p^m_{At} - c_A)s_{At}(p^m_{At}, p^m_{Bt}, \boldsymbol{n}_t) + (p^m_{Bt} - c_B)s_{Bt}(p^m_{At}, p^m_{Bt}, \boldsymbol{n}_t)]\, M_t,$$

where the superscript m denotes a multi-network firm, and (s_{At}, s_{Bt}) are the sales market shares of the two networks as in equation (1). The profit-maximization problem in period t is

$$\max_{p^m_{At}, p^m_{Bt}} \{\pi_t(p^m_{At}, p^m_{Bt}; \boldsymbol{n}_t) + \beta^d V^m_{t+1}(\boldsymbol{n}_{t+1}; \boldsymbol{n}_t, \boldsymbol{p}^m_t)\}, \tag{5}$$

subject to the transition rule in equation (2).

The multi-network firm not only faces the inter-temporal trade-off as single-network firms but also needs to internalize the competition across networks. Within period t, a higher p^m_{At} increases the firm's markup on product A, reduces the sales of product A, but increases the sales of product B. Across periods, the higher p^m_{At} leads to a greater market size, a greater network size of B, and a smaller network size of A in period $t+1$. The multi-network firm not only considers the

As explained in the introduction, the multi-network firm's pricing problem is more complex than the single-network firms' pricing problem. The three differences between the two types of firms affect the multi-network firms' optimal pricing strategy. First, the multi-network firm has an incentive to choose lower a price for the larger network due to the convexity of profits in network concentration. We show that the multi-network firm's last-period profit is convex in (n_{AT}, n_{BT}) in section A.2 of the Appendix. Second, the multi-network firm has greater market power, which allows it to set higher prices for the networks, compared with single-network firms. Third, the spillover effects across multi-network firms reduces their incentive to invest on the network size and thus has a positive impact on the prices.

Similar to the single-network firms' case, we analytically study the multi-network firm's pricing strategy for $T = 2$. In section 4, We use numerical examples to analyze the multi-network firm's strategy for $T \geq 3$. Let the optimal prices when $T = 2$ be $(p^{m*}_{A1}, p^{m*}_{B1}, p^{m*}_{A2}, p^{m*}_{B2})$. We find the following results.

Proposition 2. *(1) A has a lower price than B in the first period:* $p^{m*}_{A1} < p^{m*}_{B1}$.
(2) The price difference, $|p^{m*}_{A1} - p^{m*}_{B1}|$, *increases with the network effect strength,* γ.
(3) The difference in the two networks' market shares in the second period, $(n_{A2} - n_{B2})$, *increases with* γ.

Proof. See section A.2 of the Appendix for the proof. □

The first result of Proposition 2 says that the multi-network firm chooses a lower price for the larger network A than for B in the first period. This implies that the multi-network firm's preference for network concentration plays an important role. The intuition is that this pricing strategy can increase the network concentration, which raises consumers' willingness to pay for the large network.

The second result means that price differentiation increases as the network effect strength increases. As γ increases, it is easier to increase network concentration due to the stronger positive externality among users of the larger network A. However, this does not reduce the multi-network firm's incentive to set lower price for A. Instead, the firm chooses even greater price difference. This implies that the marginal benefit of using the differentiated prices increases with γ.

The third result implies that the market becomes more concentrated as the network effect increases. A higher γ has two positive impacts on network concentration. First, a higher γ implies that the initially larger network A can attract more consumers than network B, even when the two networks have the same price. This increases the network concentration. Second, as in the previous result, the greater network effect also increases the firm price differentiation, which further enhances the network advantage of A.

The results in Proposition 2 hold when $\gamma > 0$ and $\beta^d > 0$. If $\gamma = 0$, then A and B will have the same price because consumers have the same utility from them. The discount factor also affects the first period prices. If $\beta^d = 0$, the optimization problem is static. In this case, the Logit demand model implies that the firm will choose the same price for the two products because their cross-derivatives of prices are the same.[3]

3.3 Network Concentration

Multi-network firms invest on network concentration by sacrificing the ability to harvest on the initial network advantage of the larger network. Single-network firms also have an incentive to invest, but they each invest on their own networks. In equilibrium, the initially larger network's price is greater than the smaller network's price in each period. Although the initially larger network can keep its advantage over time, the equilibrium prices reduce the network concentration, compared with the multi-network case.

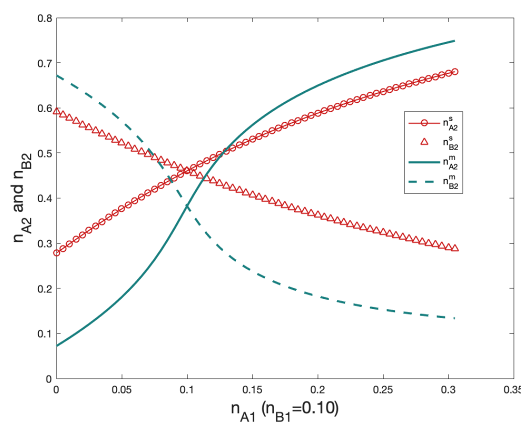


Figure 1: Network Concentration with Multi- and Single-Network Firms

We use numerical examples to show degree of network concentration. Figure 1 shows the network concentration in the single- and multi-network firm cases in a two-period model. The horizontal axis is the market share of network A in the first period,

[3]The cross derivatives are $\alpha s_{At} s_{Bt}$.

$n_{A1} \in [0, 0.35]$. The market share of B is fixed at $n_{B1} = 0.10$.[4] The vertical axis is the network market share in the beginning of the second period, (n_{A2}, n_{B2}). The curves with triangular and circle markers are n^s_{A2} and n^s_{B2} in the single-network firm case, respectively. The solid and dash curves are n^m_{A2} and n^m_{B2} in the multi-network firm case, respectively. Figure 1 shows that, when $n_{A1} = n_{B1}$, $n_{A2} = n_{B2}$ in both cases. That is, the two curves cross each other at $n_{A1} = 0.10 (= n_{B1})$. When $n_{A1} > (<) n_{B1}$, $n_{A2} > (<) n_{B2}$ in both cases, which implies that the networks can keep their initial network size advantage. When $n_{A1} = n_{B1}$, the cross point in the single-network firm case is above the cross point in the multi-network firm case. This is because the monopolist multi-network firm chooses higher prices than the single-network firms, so (n_{A2}, n_{B2}) are smaller in the multi-network firm case. However, when $n_{A1} \neq n_{B1}$, the networks in the multi-network firm case are more concentrated, compared with the single-network firm case.

Although Figure 1 only considers a monopolist multi-network firm, the results above will hold when one considers competing multi-network firms. As section 5 will show, when there are competing multi-network firms, they all choose lower prices for the initially larger network. Furthermore, the multi-network firms' prices for all networks go down when competition increases. Therefore, the network concentration will be greater when more competing multi-network firms enter the market.

4 Numerical Examples for $T > 2$

In a T-period dynamic game, the single- and multi-network firms still face the same trade-offs as in the two-period model. In period T, the multi-network firm faces a static pricing problem, and the single-network firms play a static pricing game. In period $t < T$, the multi-network firm has an incentive to increase network concentration, since

[4]The parameter values are $\alpha = 2$, $\gamma = 8$, $\beta = 0.96$, $\delta_a = \delta_b = 0$, $c_a = c_b = 0$.

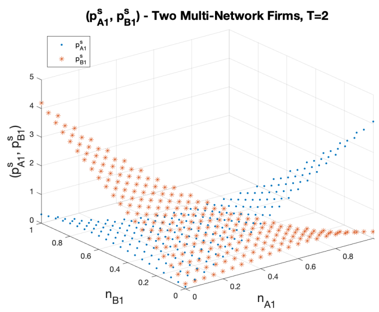
the profit function in the last period is convex in $(n_{AT}, n_{BT}$. Thus, the firm chooses a lower price for the larger network, while a single-network firm chooses a higher price if the firm has a larger network size. In this section, we use numerical example to show the pricing strategies of multi- and single-network firms for $T \geq 2$.

Figure 2 shows the equilibrium prices of two single-network firms in a finite-horizon game for $T \in \{2, 3, 4\}$.[5] In each subfigure, the x-axis is n_{At} ($t \leq T$), the y-axis is n_{Bt}, and the z-axis shows the prices, (p^s_{At}, p^s_{Bt}) in period t. The scatter plot with blue dot markers are p^s_{At}, and the scatter plot with red asteroid markers are p^s_{Bt}. Subfigures (a) and (b) are the prices in the first and second periods when $T = 2$. When $n_{At} > n_{Bt}$, the prices satisfy $p^s_{At} > p^s_{Bt}$. This finding is in line with the results in proposition 1.

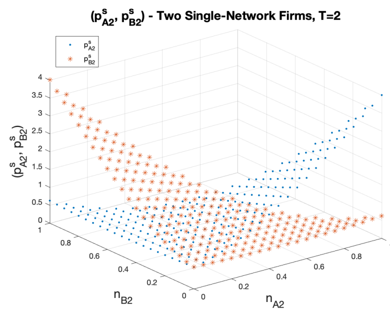
Subfigures (c), (d), and (e) in Figure 2 plot the prices in each of the three periods when $T = 3$. Subfigures (f), (g), and (h) plot the prices in the last three periods when $T = 4$. These figures also show that the equilibrium price of the larger network is always greater than the price of the smaller network in each period when $T \in \{3, 4\}$. This indicates that the single-network firms' pricing strategies in Proposition 1 holds when $T > 2$.

[5] I first generate the grid values for n_{At} and n_{Bt}. To solve for the prices in period T, we use the FOCs in period T. To solve for prices in period $T - 1$, we first ....

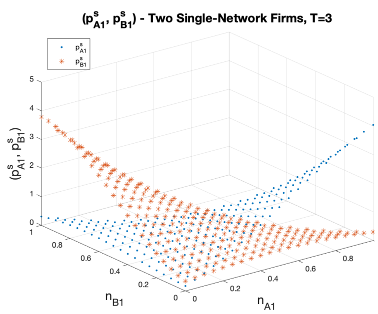
Figure 2: Single-Network Firms' Prices (T=2,3,4)



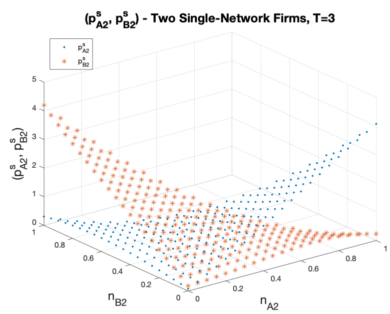
(a) (p^s_{A1}, p^s_{B1}), T=2



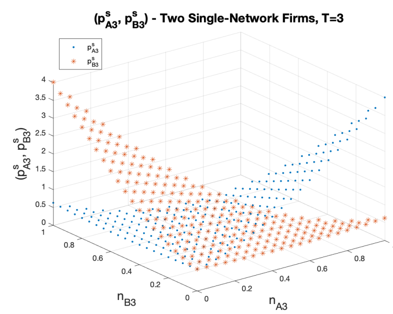
(b) (p^s_{A2}, p^s_{B2}), T=2



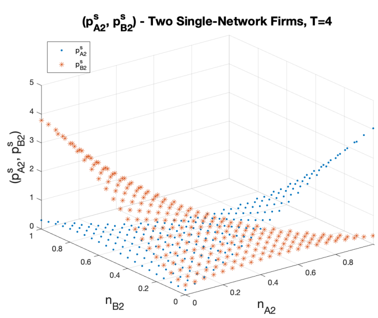
(c) (p^s_{A1}, p^s_{B1}), T=3



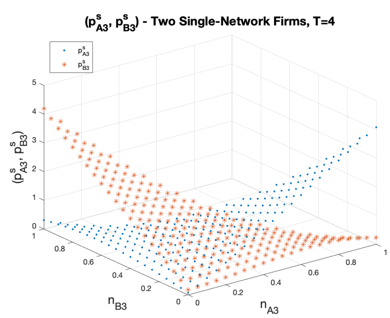
(d) (p^s_{A2}, p^s_{B2}), T=3



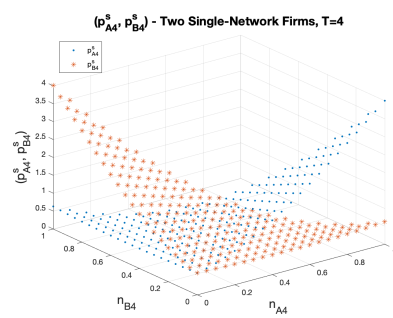
(e) (p^s_{A3}, p^s_{B3}), T=3



(f) (p^s_{A2}, p^s_{B2}), T=4

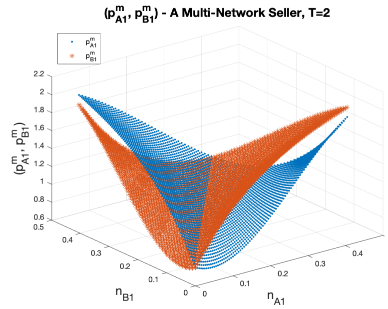


(g) (p^s_{A3}, p^s_{B3}), T=4

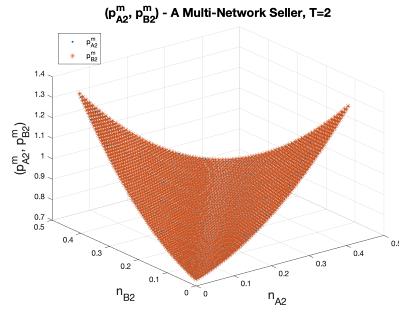


(h) (p^s_{A4}, p^s_{B4}), T=4

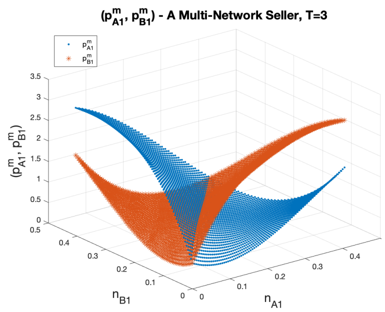
Figure 3: A Multi-Network Firm's Prices (T=2,3,4)



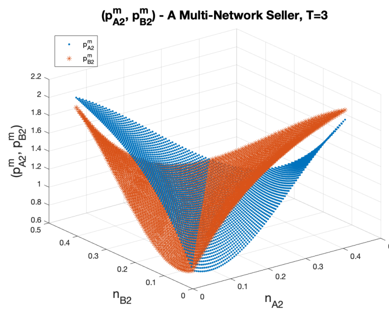
(a) (p^m_{A1}, p^m_{B1}), T=2



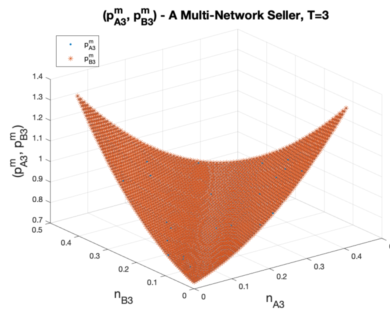
(b) (p^m_{A2}, p^m_{B2}), T=2



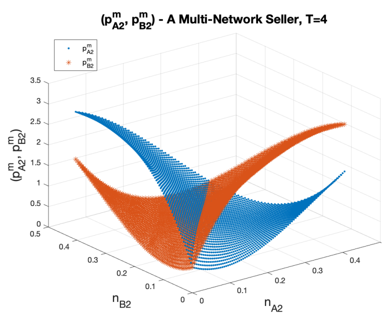
(c) (p^m_{A1}, p^m_{B1}), T=3



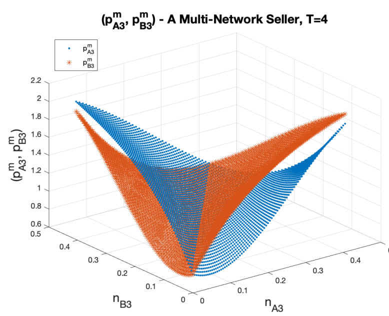
(d) (p^m_{A2}, p^m_{B2}), T=3



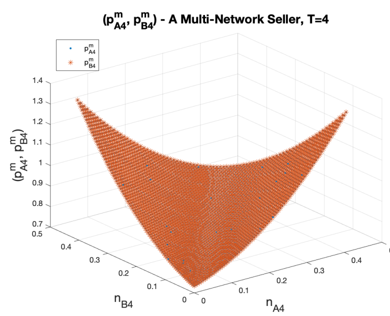
(e) (p^m_{A3}, p^m_{B3}), T=3



(f) (p^m_{A2}, p^m_{B2}), T=4



(g) (p^m_{A3}, p^m_{B3}), T=4



(h) (p^m_{A4}, p^m_{B4}), T=4

Figure 3 shows the equilibrium prices of a monopolist multi-network firm for $T \in \{2,3,4\}$. In each subfigure, the x-axis is n_{At}, the y-axis is n_{Bt}, and the z-axis shows the prices, (p^m_{At}, p^m_{Bt}) in period t. As in Figure 2, the scatter plot with blue dot markers are p^m_{At}, and the scatter plot with red asteroid markers are p^m_{Bt}. Subfigures (a) and (b) are the prices in the first and second periods when $T = 2$. Subfigure (a) shows that, when $n_{A1} > n_{B1}$, the prices satisfy $p^m_{A1} < p^m_{B1}$. That is, the multi-network firm chooses a lower price for the larger network in the first period. Subfigure (b) shows that $p^m_{A2} = p^m_{B2}$ in the second period. These findings are consistent with the results in proposition 2.

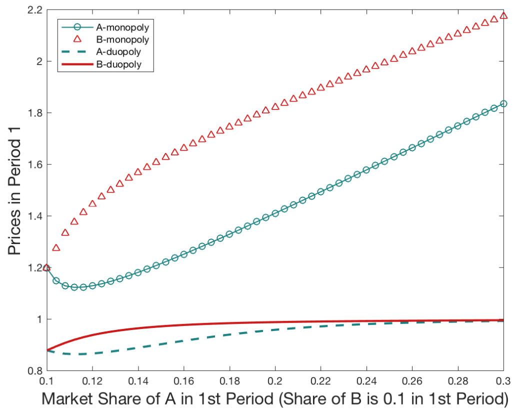
Subfigures (c), (d), and (e) plot the prices each of the three periods when $T = 3$. Subfigures (f), (g), and (h) are the prices in the last three periods when $T = 4$. Similar to the results for $T = 2$, the prices satisfy $p^m_{At} < p^m_{Bt}$ when $n_{At} > n_{Bt}$ for $t < T$, and vice versa. In the last period, the multi-network firm chooses the same price for the two networks, $p^m_{AT} = p^m_{BT}$. This indicates that the multi-network firm starts to build network concentration from the first period. Therefore, the multi-network firm chooses a lower price for the larger network when $t < T$, when $T \in \{3, 4\}$. This indicates that the price pattern in Proposition 2 holds when $T > 2$.

5 Competing Multi-Network firms

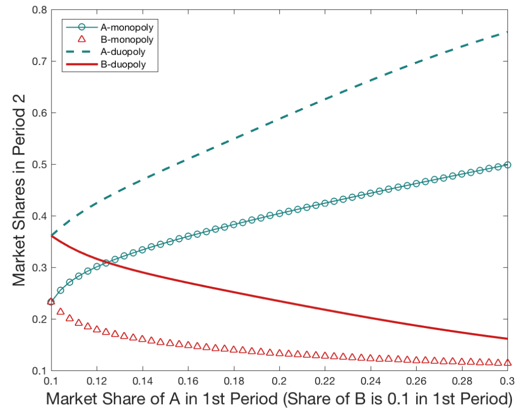
Competition among multi-network firms may affect the results in Proposition 2. Competition has two impacts on multi-network firms' prices. First, competition drives down the prices. As the number of multi-network firms increases, the equilibrium prices for all network will decrease. This is the same to the impact of competition on prices in markets without network effects. Second, all competing multi-network firms have the incentive to choose lower prices for the initially larger network before the last period. That is, multi-network firms will not invest on different networks by choosing lower prices for different networks. To see this, consider a two-period duopoly model with two identical multi-network firms, 1 and 2. Suppose that firm 1 chooses a lower price for network A in the first period. Given firm 1's pricing strategy, firm 2's best response is to choose the same strategy. If instead, firm 2 chooses a lower price for B, then its profits will be lower than if it sets the same prices as firm 1. When the two firms choose the same prices, they will obtain the same profits. However, when firm 2 chooses a lower price for network B, then firm 1 will attract more consumers due to the network advantage of network A. In this case, firm 2's profits will be less than when it can split the market with firm 1.

To show the impacts of competition on the prices, we solve numerical examples with two identical multi-network firms who play a two-period dynamic pricing game. The equilibrium prices show the following results. First, the two firms choose the same prices for the two networks. This occurs because the two firms are identical. Second, both firms set a lower price for A than for B in the first period, which is the same as in the monopoly case. This indicates that they do not invest on different networks. Third, competition leads to lower equilibrium prices in the duopoly case than in the monopoly case.

I show these results in Figure 4. Figure 4a compares $(p_{A1}^{m*}, p_{B1}^{m*})$ under the monopoly and duopoly cases. The horizontal axis is n_{A1}, ranging from 0.1 to 0.3. We fix the initial share of network B at 0.1 in both cases. The curves with circle and triangle markers are p_{A1}^{m*} and p_{B1}^{m*} in the monopoly case, respectively. The dashed and solid curves are p_{A1}^{m*} and p_{B1}^{m*} in the duopoly case, respectively. This figure confirms the findings above. First, in both cases, network A has a lower price than network B, $p_{A1}^{m*} < p_{B1}^{m*}$. Second, in the duopoly case, the two firms choose the same pricing strategy. Third, the equilibrium prices for both networks are lower in the duopoly case than in the monopoly case.



(a) Prices under Monopoly and Duopoly



(b) Network Size under Monopoly and Duopoly

Figure 4: A Monopoly Multi-network firm vs. Duopoly Multi-network firms

Figure 4b plots n_{A2} and n_{B2} under the two cases. The horizontal axis is n_{A1}. The curves with circle and triangle markers are n_{A2} and n_{B2} in the monopoly case, respectively. The dashed and solid curves are n_{A2} and n_{B2} in the duopoly case, respectively. We find the following results. First, in both cases, network A has a larger share than network B in the second period. Second, both networks have higher market shares, (n_{A2}, n_{B2}), in the duopoly case than in the monopoly case. This is because both networks have lower prices in the duopoly case. Third, as n_{A1} increases, n_{A2} increases and n_{B2} decreases. This implies that, as the initial network advantage of network A goes up, network B attracts fewer consumers in the first period and the network concentration increases.

6 Application: Telecom Carriers' Pricing of Smartphones

6.1 The OS Network Effect and Multi-OS Telecom Carriers

To see the multi-network firms' pricing strategies in practice, we analyze the telecom carriers' pricing strategies for smartphones in the U.S.. Smartphone OSs are subject to direct and indirect network effects. An indirect network effect exists because developers prefer to launch apps on OSs with more users, and users are more likely to adopt OSs with more apps. In addition, a direct network effect also exists due to the benefits of adopting the same OS with friends and family (e.g., low learning cost of using smartphones with the same OS, ease of sending files and photos, and sharing of app and music purchases). Bresnahan, Orsini, and Yin (2014) find a positive feedback loop of the indirect network effect for smartphones. By estimating a structural model of consumers' demand for smartphones, Luo (2022) finds that the OS network effect is positive and significant.

Given the OS network effects, we use the monthly carrier-OS-level data in Luo (2022) to show the pricing strategy of the telecom carriers in this section. The sample period is August 2011 to September 2013. During this sample period, the leading carriers were Verizon, AT&T, Sprint, and T-Mobile, and there were four major smartphone OSs, iOS, Android, Blackberry, and Windows Phone. Each carrier sold multiple OSs' smartphone models.

During the sample period, the telecom carriers did not sell the smartphones at the manufacturers' retail prices. Instead, they used the two-year contract mode in order to attract more consumers. If consumers signed two-year wireless service contracts, they could receive significant discounts on smartphones. For example, the iPhone 5's retail price was $649, but a consumer only had to pay $199 to the carriers if he signed a contract. The two-year contract mode was very successful. According to the US Wireless Industry Overview 2011, more than 78% of mobile phone users were on two-year contracts. The percentage was even higher for smartphones which were significantly more expensive than feature phones without the contracts. The discounts on the smartphones varied across carriers, OSs, and models. By choosing the contract discounts for the smartphones, the carriers were able to choose differentiated discounts across OSs to maximize long-run profits.

The data covers four carriers (Verizon, AT&T, Sprint, and T-Mobile) and four OSs (iOS, Android, Blackberry, and Windows Phone). This dataset includes the smartphone prices with two-year contracts, the manufacturers' smartphone prices, the cumulative number of OS users (network size), and the smartphone characteristics. We aggregate these variables to the carrier-OS level. In each month, there are up to 16 carrier-OS product combinations since Sprint and T-Mobile only started selling iPhones later in the sample period. The total number of carrier-OS-month observations is 355.

6.2 Multi-OS Carriers' Pricing Strategy

To analyze the carriers' pricing strategy, we regress the two-year contract phone prices on the OS network size. In each month, the OS network size is the number of OS users at the end of the previous month. Other control variables in the regression include the manufacturers' retail price, phone characteristics, number of OS version updates, OS dummies, and carrier dummies. The results are in Table 1. Column (1) shows the ordinary least squares (OLS) results. The coefficient for OS size indicates a negative correlation between the lagged OS network sizes and the carriers' smartphone prices. This negative correlation means that the carriers choose relative lower prices for the larger OSs, which is consistent with the multi-network firms' pricing strategy in Proposition 2 and Figure 4a. Since Apple produces all the iOS models, the negative correlation might coincide with Apple reducing prices of old models over time and iOS's market share increasing. Thus, we drop the iOS observations in column (2), and the negative correlation still exists.

The OS network size is endogenous if the error term in the OLS regression has a serial correlation. To address this issue, we use the current monthly cumulative number of apps of each OS as the IV for the network size. Since it takes months to develop an app, we assume that the number of apps is not correlated with the current unobserved quality. This IV passes the commonly used weak-IV test.[6] Columns (3) and (4) of Table 1 show the IV regression results. Column (3) uses all the observations, and column (4) drops the iOS observations. The results in these two columns still show a negative correlation between the carriers' smartphone prices and the OS network sizes. Therefore, the multi-OS carriers choose lower prices for the larger OSs than for the smaller OSs.

Table 1: Regression of Carriers' Contract Phone Prices on OS Network Sizes (\$100)

VARIABLES	(1) Price OLS	(2) Price OLS (no iOS)	(3) Price IV	(4) Price IV (no iOS)
OS size	-0.876**	-0.974*	-1.736***	-2.419***
	(0.435)	(0.580)	(0.566)	(0.835)
Manufacturer price	0.409***	0.322***	0.406***	0.316***
	(0.060)	(0.071)	(0.058)	(0.069)
Observations	355	277	355	277
R-squared	0.829	0.546	0.827	0.535

Standard errors are in parentheses.
*** p<0.01, ** p<0.05, * p<0.1

This regression uses the monthly average carrier-OS-level prices and the one-month lagged OS network sizes from August 2011 to October 2013. We also control for phone characteristics, OS-specific version updates, OS dummies, and carrier dummies in all specifications.

[6]The value of the F-statistic for the first-stage regression is 2239.26, which is significantly larger than the rule-of-thumb value (10) for the weak-IV test.

7 Conclusion

This paper aims to investigate the dynamic pricing strategy of multi-network firms. While many industries (e.g., smartphone, television, and automobile) feature multi-network firms, the literature has paid little attention to their dynamic pricing strategy. This paper fills the gap by studying the multi-network firms' pricing strategies.

Multi-network firms face very different dynamic pricing problems from single-network firms. First, they have a preference for network concentration. This is because their profits increase with network concentration. This preference gives them an incentive to choose lower prices for the initially larger networks. Second, they have greater market power than single-network firms because they sell multiple networks' products and thus can internalize the competition across networks. This market power effect has a positive on their prices for all networks. Third, a spillover effect exists across multi-network firms when they sell the same networks' products. This reduces each multi-network firm's incentive to invest on the network sizes with low prices. Thus, the spillover effect has a positive impact on all networks' prices. By estimating a structural model and analyzing counterfactual scenarios, Luo (2022) empirically shows these three impacts in the smartphone industry.

To analyze the multi-network firms' pricing strategy theoretically, we set up a finite-horizon theoretical model with two networks to compare the pricing strategies of multi- and single-network firms. Instead of using the Hotelling demand model or the demand setup in Cabral (2011), we use the discrete choice demand model. This model not only allows consumers to choose the outside option (not joining any network) in each period but also allows the market size to change across periods. These features make the discrete choice model a more realistic setting than the demand models mentioned above.

By solving the firms' pricing problems and games, we find the following results.

First, with single-network firms, the equilibrium price of the initially larger network is greater then the price of the smaller network in each period. This result is similar to that in Cabral (2011). However, the demand model is very different in this paper. In addition, we show that, although the larger network has higher prices, it can keep the network size advantage through out the game. Second, multi-network firms set lower prices for the products with larger networks than those with smaller networks before the last period, which is the opposite of the single-network firms' pricing pattern. This implies that the multi-network firms' preference for network concentration is playing an important role. This strategy enhances the network advantage of the initially larger network and increases the firms' long-run profits. Third, compared with the single-network firm case, the market becomes more concentrated in the multi-network case.

The multi-network firms' pricing strategy is present in the smartphone industry, where the smartphone OS network effect exists, and the telecom carriers act like multi-network firms. The carriers set prices for two-year contract smartphones by choosing contract discounts for smartphones. The amount of discount varied across smartphones and OSs. We use monthly carrier-OS-level data from 2011 to 2013 to show the multi-OS carriers' pricing strategies of the smartphones. We regress the carriers' contract prices for smartphones on the lagged OS network sizes and deal with the endogeneity in the OS network size with an IV. The results show that the multi-OS carriers choose lower prices for the larger OSs (iOS and Android) than for the smaller OSs (Blackberry and Windows Phone). This finding is consistent with the multi-network firms' pricing strategy in the theoretical model.

With the emergence of new industries that rely on the consumer networks (e.g., online shopping, video streaming, electric vehicles, and delivery services), more studies on the impacts of network effects on the firms' prices and market concentration will be very important. The results of this paper shed light on different types of firms' pricing strategies and market concentration in industries with network effects, especially in industries with multi-network firms.

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Appendix

A Proofs in the Two-OS, Two-Period Model

Lemma 1. *Given that the two models have the same unit cost, the multi-network firm chooses the same price for A and B in the second period:* $p^{*c}_{A2} = p^{*c}_{B2}$.

Proof. Let the unit cost of the two models be c. The firm's profit in the 2nd period is:

$$\begin{aligned}\pi_2(p_{A1}, p_{B1}, p_{A2}, p_{B2}) &= \sum_{j=A,B} (p_{j2} - c) M_2 s_{j2} \\ &= \sum_{j=A,B} (p_{j2} - c) M_2 \frac{\mathrm{e}^{(\delta_j + \gamma n_{j2} - \alpha p_{j2})}}{1 + \sum_{k=A,B} \mathrm{e}^{(\delta_k + \gamma n_{k2} - \alpha p_{k2})}}.\end{aligned}$$

The FOC with respect to p_{A2} is:

$$s_{A2} - \alpha(p^*_{A2} - c)s_{A2}(1 - s_{A2}) + \alpha(p^*_{B2} - c)s_{A2}s_{B2} = 0, \tag{A.1}$$

which is equivalent to:

$$1 - \alpha(p^*_{A2} - c) + \alpha(p^*_{A2} - c)s_{A2} + \alpha(p^*_{B2} - c)s_{B2} = 0. \tag{A.2}$$

Similarly for B, we have:

$$1 - \alpha(p^*_{B2} - c) + \alpha(p^*_{A2} - c)s_{A2} + \alpha(p^*_{B2} - c)s_{B2} = 0. \tag{A.3}$$

Compare equations (A.2) and (A.3), we get $p^*_{A2} = p^*_{B2}$. □

A.1 Proof of Proposition 1

Proof. I solve the two-period game backwards. Let the price in the 2nd period of the two models be p^*_2. Then plug p^*_2 and $c = 0$ into equation (A.1). We get:

$$1 - \alpha p^*_2(1 - s_{A2} - s_{B2}) = 0.$$

Plug in the sales market share equations and rearrange the terms. We get:

$$\alpha p^*_2 - 1 = \mathrm{e}^{(\delta_A + \gamma n_{A2} - \alpha p^*_2)} + \mathrm{e}^{(\delta_B + \gamma n_{B2} - \alpha p^*_2)}. \tag{A.4}$$

The total differentiation of (A.4) is:

$$\alpha \frac{\partial p^*_2}{\partial n_{A2}} = \mathrm{e}^{(\delta_A + \gamma n_{A2} - \alpha p^*_2)}(\gamma - \alpha \frac{\partial p^*_2}{\partial n_{A2}}) - \alpha \mathrm{e}^{(\delta_B + \gamma n_{B2} - \alpha p^*_2)} \frac{\partial p^*_2}{\partial n_{A2}}.$$

Then we can solve for $\frac{\partial p^*_2}{\partial n_{A2}}$:

$$\frac{\partial p^*_2}{\partial n_{A2}} = \frac{\gamma}{\alpha} s_{A2}. \tag{A.5}$$

Similarly for $\frac{\partial p^*_2}{\partial n_{B2}}$, we have:

$$\frac{\partial p^*_2}{\partial n_{B2}} = \frac{\gamma}{\alpha} s_{B2}. \tag{A.6}$$

Then the profit in the 2nd period is:

$$\begin{aligned}\pi_2 &= \sum_{j=A,B} p_{j2} M_2 s_{j2} \\ &= p_2^* M_2 (s_{A2} + s_{B2}) \\ &= p_2^* M_2 (1 - \frac{1}{1 + e^{(\delta_A + \gamma n_{A2} - \alpha p_2^*)} + e^{(\delta_B + \gamma n_{B2} - \alpha p_2^*)}}) \\ &= p_2^* M_2 (1 - \frac{1}{\alpha p_2^*}) \\ &= \frac{M_2}{\alpha}(\alpha p_2^* - 1).\end{aligned} \tag{A.7}$$

Then the maximization problem in the first period is:

$$\begin{aligned}&\max_{p_{A1}, p_{B1}} \pi_1(p_{A1}, p_{B1}) + \beta \pi_2(p_{A1}, p_{B1}, p_2^*(p_{A1}, p_{B1})) \\ =&\max_{p_{A1}, p_{B1}} \sum_{j=A,B} p_{j1} M_1 s_{j1} + \frac{\beta}{\alpha}(\alpha p_2^*(p_{A1}^*, p_{B1}^*) - 1) M_2 \\ =&\max_{p_{A1}, p_{B1}} \sum_{j=A,B} p_{j1} M_1 s_{j1} + \beta(p_2^*(p_{A1}, p_{B1}) - \frac{1}{\alpha}) M_1 s_{01}.\end{aligned} \tag{A.8}$$

in which $s_{01} = 1 - s_{A1} - s_{B1}$ (0 means the outside option) and those who didn't buy any smartphone in the first period enter the second period, $M_2 = M_1 s_{01}$. Then the FOC with respect to p_{A1} is:

$$\begin{aligned}0 =& s_{A1} - \alpha p_{A1}^* s_{A1}(1 - s_{A1}) + \alpha p_{B1}^* s_{A1} s_{B1} \\ &+ \alpha\beta(p_2^*(p_{A1}, p_{B1}) - \frac{1}{\alpha}) s_{A1} s_{01} + \beta \frac{\partial p_2^*}{\partial p_{A1}} s_{01},\end{aligned} \tag{A.9}$$

in which the partial derivative of price p_2^* with respect to 1st period price p_{A1} is:

$$\begin{aligned}\frac{\partial p_2^*}{\partial p_{A1}} &= \frac{\partial p_2^*}{\partial n_{A2}} \frac{\partial n_{A2}}{\partial p_{A1}} + \frac{\partial p_2^*}{\partial n_{B2}} \frac{\partial n_{B2}}{\partial p_{A1}} \\ &= \frac{\partial p_2^*}{\partial n_{A2}} M_1(-\alpha) s_{A1}(1 - s_{A1}) + \frac{\partial p_2^*}{\partial n_{B2}} M_1 \alpha s_{A1} s_{B1}.\end{aligned}$$

Plug $\frac{\partial p_2^*}{\partial p_{A1}}$ into (A.9), we get:

$$\begin{aligned}&1 - \alpha p_{A1}^* + \alpha p_{A1}^* s_{A1} + \alpha p_{B1}^* s_{B1} + \beta(p_2^* - \frac{1}{\alpha}) \alpha s_{01} \\ &+ \beta M_1 s_{01}[-\alpha \frac{\partial p_2^*}{\partial n_{A2}}(1 - s_{A1}) + \alpha \frac{\partial p_2^*}{\partial n_{B2}} s_{B1}] = 0.\end{aligned} \tag{A.10}$$

Similarly for OS B, the first-order condition of profit with respect to price p_{B1} gives the following equation:

$$\begin{aligned}&1-\alpha p^*_{B1}+\alpha p^*_{B1}s_{B1}+\alpha p^*_{A1}s_{A1}+\beta(p^*_2-\frac{1}{\alpha})\alpha s_{01}\\&+\beta M_1 s_{01}[-\alpha\frac{\partial p^*_2}{\partial n_{B2}}(1-s_{B1})+\alpha\frac{\partial p^*_2}{\partial n_{A2}}s_{A1}]=0.\end{aligned}\tag{A.11}$$

Then by comparing equations (A.10) and (A.11), we get:

$$\begin{aligned}&-\alpha p^*_{A1}+\beta M_1 s_{01}[-\alpha\frac{\partial p^*_2}{\partial n_{A2}}(1-s_{A1})+\alpha\frac{\partial p^*_2}{\partial n_{B2}}s_{B1}]\\=&-\alpha p^*_{B1}+\beta M_1 s_{01}[-\alpha\frac{\partial p^*_2}{\partial n_{B2}}(1-s_{B1})+\alpha\frac{\partial p^*_2}{\partial n_{A2}}s_{A1}].\end{aligned}$$

Plug in the partial derivatives in (A.5) and (A.6), we get:

$$p^*_{A1}-p^*_{B1}=\frac{\beta\gamma}{\alpha}M_1 s_{01}(s_{B2}-s_{A2}).\tag{A.12}$$

Next we need to prove $p^*_{A1}<p^*_{B1}$ if $n_{A1}>n_{B1}$. That is, the 1st period price for the large OS is lower than that for the small OS. Notice that $s_{B2}<s_{A2}\Leftrightarrow n_{B2}<n_{A2}$, given $p^*_{B2}=p^*_{A2}$ and $\delta_A=\delta_B=\delta$. Next We discuss the three cases of possible relationship between p^*_{A1} and p^*_{B1} to prove that $p^*_{A1}<p^*_{B1}$ must hold to maximize profit.

First, suppose $p^*_{A1}=p^*_{B1}$. Then $n_{B2}<n_{A2}$. Because $n_{j2}=n_{j1}+M_1 s_{j1}$, $n_{A1}>n_{B1}$, and $s_{A1}=s_{B1}$. But this means (A.12) is violated because $s_{B2}-s_{A2}<0$.

Second, suppose $p^*_{A1}>p^*_{B1}$. If this is true, then (A.12) implies that $s_{B2}>s_{A2}$, which means that $n_{B2}>n_{A2}$. That is, the initial OS network advantage of A is reversed in the 2nd period due to high price of A. Since $n_{j2}=n_{j1}+M_1 s_{j1}$, $n_{B2}>n_{A2}$ implies that $s_{B1}>s_{A1}$. That is, the sales share of B is higher than that of A in the first period. Use the equation of sales market shares, we get:

$$\mathrm{e}^{(\delta+\gamma n_{B1}-\alpha p^*_{B1})}>\mathrm{e}^{(\delta+\gamma n_{A1}-\alpha p^*_{A1})}.\tag{A.13}$$

Let $(p^*_{A1},p^*_{B1},p^*_2)$ bet the profit maximization prices in the two periods. Then the firm's total profit is:

$$\Pi^*=M_1\frac{p^*_{A1}\mathrm{e}^{(\delta+\gamma n_{A1}-\alpha p^*_{A1})}+p^*_{B1}\mathrm{e}^{(\delta+\gamma n_{B1}-\alpha p^*_{B1})}+\beta/\alpha(\alpha p^*_2-1)}{1+\mathrm{e}^{(\delta+\gamma n_{A1}-\alpha p^*_{A1})}+\mathrm{e}^{(\delta+\gamma n_{B1}-\alpha p^*_{B1})}}.$$

Now consider a different price plan for the two periods (p'_{A1},p'_{B1},p^*_2), in which:

$$\gamma n_{A1}-\alpha p'_{A1}=\gamma n_{B1}-\alpha p^*_{B1},$$

$$\gamma n_{B1}-\alpha p'_{B1}=\gamma n_{A1}-\alpha p^*_{A1}.$$

Then $p'_{A1}=\frac{\gamma n_{A1}-\gamma n_{B1}+\alpha p^*_{B1}}{\alpha}$ and $p'_{B1}=\frac{\gamma n_{B1}-\gamma n_{A1}+\alpha p^*_{A1}}{\alpha}$. The firm's total profit with this price plan is:

$$\Pi' = M_1 \frac{p'_{A1}e^{(\delta+\gamma n_{A1}-\alpha p'_{A1})} + p'_{B1}e^{(\delta+\gamma n_{B1}-\alpha p'_{B1})} + \beta/\alpha(\alpha p_2^* - 1)}{1 + e^{(\delta+\gamma n_{A1}-\alpha p'_{A1})} + e^{(\delta+\gamma n_{B1}-\alpha p'_{B1})}}$$
$$= M_1 \frac{p'_{B1}e^{(\delta+\gamma n_{A1}-\alpha p^*_{A1})} + p'_{A1}e^{(\delta+\gamma n_{B1}-\alpha p^*_{B1})} + \beta/\alpha(\alpha p_2^* - 1)}{1 + e^{(\delta+\gamma n_{A1}-\alpha p^*_{A1})} + e^{(\delta+\gamma n_{B1}-\alpha p^*_{B1})}}.$$

Take the difference of the two profits with the two price plans, we have:

$$\Pi' - \Pi^* = M_1 \frac{\gamma/\alpha(n_{A1} - n_{B1})(e^{(\delta+\gamma n_{B1}-\alpha p^*_{B1})} - e^{(\delta+\gamma n_{A1}-\alpha p^*_{A1})})}{1 + e^{(\delta+\gamma n_{A1}-\alpha p^*_{A1})} + e^{(\delta+\gamma n_{B1}-\alpha p^*_{B1})}}.$$

The according to (A.13), we know that $\Pi' > \Pi^*$. Hence, there exists another price plan that leads to higher profit than $(p^*_{A1}, p^*_{B1}, p^*_2)$, when $p^*_{A1} > p^*_{B1}$. Therefore $p^*_{A1} > p^*_{B1}$ can not be the profit maximization solution.

Therefore, the firm's profit maximization prices in the first period must satisfy $p^*_{A1} < p^*_{B1}$. This implies that $n_{A2} > n_{B2}$ and thus $s_{A2} > s_{B2}$. Then we have:

$$p^*_{A1} - p^*_{B1} = \frac{\beta\gamma}{\alpha} M_1 s_{01}(s^{os}_{B2} - s^{os}_{A2}),$$

$$n_{A2} - n_{B2} = n_{A1} - n_{B1} + M_1(s_{A1} - s_{B1}).$$

Based on these two equations, we have the following conclusions:

(1) The optimal price of A is lower than that of B in the first period: $p^*_{A1} < p^*_{B1}$.

(2) The price gap $|p^*_{A1} - p^*_{B1}|$ between the two models increase as the OS network effect becomes stronger (γ increases).

(3) The OS market share difference $(n_{A2} - n_{B2})$ increases in the OS network effect γ.

□

A.2 Proof of Proposition 2

Proof. This proof of the single-network firms' dynamic pricing game has three steps. The first step shows that the price of the larger OS network is higher in the second period. The second step shows that the price of the larger OS network is higher in the first period. The third step shows that the larger operating system keeps its advantage in the second period. Combining the three steps, Proposition 2 is proved.

1. Step 1. This steps shows that, if $n_{A2} > n_{B2}$ at the beginning of the second period, then $p^s_{A2} > p^s_{B2}$. firm j's problem in the second period is:

$$\underset{p^s_{j2}}{max}\{\pi^s_{j2}(p^s_{j2}, p^s_{-j2})\} = p^s_{j2} s_{j2} M_2$$
$$= p^s_{j2} \frac{e^{(\delta_j+\gamma n_{j2}-\alpha p^s_{j2})}}{1 + \sum_{k=A,B} e^{(\delta_k+\gamma n_{k1}-\alpha p^s_{k2})}} M_2.$$

Then the FOC w.r.t. price is:

$$s_{j2} + p_{j2}^{*s}(-\alpha s_{j2} + \alpha s_{j2}^2) = 0,$$

which is equivalent to the following equation since $s_{j2} > 0$:

$$p_{j2}^{*s} = \frac{1}{\alpha(1 - s_{j2})}. \tag{A.14}$$

By comparing the equations (A.14) for model A and B, we have the following equation:

$$\frac{p_{A2}^{*s}}{p_{B2}^{*s}} = \frac{1 - s_{B2}}{1 - s_{A2}}. \tag{A.15}$$

From equation (A.15) and the assumption that $n_{A2} > n_{B2}$, the result $p_{A2}^{*s} > p_{B2}^{*s}$ holds. The proof is as follows. Suppose that $p_{A2}^{*s} \leq p_{B2}^{*s}$. This implies that model A not only has larger OS network size ($n_{A2} > n_{B2}$), but also has a lower price in the second period. then $s_{A2} > s_{B2}$. So the LHS (left hand side) of equation (A.15) is less than 1 but the RHS (right hand side) is greater than 1. This contradiction shows that $p_{A2}^{*s} > p_{B2}^{*s}$ when $n_{A2} > n_{B2}$. That is, the model with larger OS network size at the beginning of the second period has higher price in the second period.

2. Step 2. This step is to show that if $n_{A1} > n_{B1}$ and $n_{A2} > n_{B2}$, then $p_{A1}^{*s} > p_{B1}^{*s}$. That is, the optimal price of the larger OS model is higher in the first period.

   From the Step 1, the maximum profit in the second period for j is:

$$\begin{aligned}
\pi_{j2}^{*s} &= p_{j2}^{*s} s_{j2} M_2 \\
&= \frac{s_{j2}}{\alpha(1 - s_{j2})}(1 - n_{A2} - n_{B2}) \\
&= \frac{s_{j2}}{\alpha(1 - s_{j2})} M_1 s_{01},
\end{aligned}$$

   in which s_{01} is the market share of the outside option in the first period. Then firm j's profit maximization problem in the first period is:

$$\begin{aligned}
&\max_{p_{j1}^s}\{\pi_{j1}^s(p_{j1}^s, p_{-j1}^s) + \beta \pi_{j2}^{*s}\} \\
&= p_{j1}^s s_{j1} M_1 + \frac{\beta}{\alpha}\frac{s_{j2}}{1 - s_{j2}} s_{01} M_1 \\
&= p_{j2}^s \frac{\mathrm{e}^{(\delta_j + \gamma n_{j1} - \alpha p_{j1}^s)}}{1 + \sum_{k=A,B} \mathrm{e}^{(\delta_k + \gamma n_{k1} - \alpha p_{k1}^s)}} M_1 + \frac{\beta}{\alpha}\frac{s_{j2}}{1 - s_{j2}} s_{01} M_1.
\end{aligned}$$

Then the FOC w.r.t. p^s_{j1} is:

$$
\begin{aligned}
&s_{j1} - \alpha s_{j1}(1-s_{j1}) + \beta s_{01} s_{j1} \frac{s_{j2}}{1-s_{j2}} \\
&+ \frac{\beta}{\alpha} s_{01} \frac{1}{(1-s_{j2})^2} \frac{\partial s_{j2}}{\partial s_{j1}} \frac{\partial s_{j1}}{\partial p^s_{j1}} = 0 \qquad .
\end{aligned} \tag{A.16}
$$

Using the definition of s_{j2} and s_{j1}, we get the following partial derivatives:

$$
\begin{aligned}
\frac{\partial s_{j2}}{\partial s_{j1}} &= \gamma s_{j2}(1-s_{j2}), \\
\frac{\partial s_{j1}}{\partial p^s_{j1}} &= -M_1 \alpha s_{j1}(1-s_{j1}).
\end{aligned} \tag{A.17}
$$

Plug equations in (A.17) into equation (A.16) and rearrange the terms, we get the following equation:

$$
1 + \beta s_{01} \frac{s_{j2}}{1-s_{j2}} = (1-s_{j1})(\alpha p^{*s}_{j1} - \beta M_1 s_{01} \frac{\gamma s_{j2}}{1-s_{j2}}). \tag{A.18}
$$

Equation (A.18) can be applied to both model A and model B, then by comparing the two sides for the two models, we get:

$$
\frac{1 + \beta s_{01}\frac{s_{A2}}{1-s_{A2}}}{1 + \beta s_{01}\frac{s_{B2}}{1-s_{B2}}} = \underbrace{\frac{1-s_{A1}}{1-s_{B1}}}_{R1} * \underbrace{\frac{\alpha p^{*s}_{A1} - \beta M_1 s_{01}\frac{\gamma s_{A2}}{1-s_{A2}}}{\alpha p^{*s}_{B1} - \beta M_1 s_{01}\frac{\gamma s_{B2}}{1-s_{B2}}}}_{R2}. \tag{A.19}
$$

Given the assumption that $n_{A2} > n_{B2}$, it is shown in Step 1 that $s_{A2} > s_{B2}$. So for the equation (A.19), $LHS > 1$. Next, we show that $p^{*s}_{A1} > p^{*s}_{B1}$.

Suppose $p^{*s}_{A1} \le p^{*s}_{B1}$, then $s_{A1} > s_{B1}$ because model A not only has the OS network advantage but also lower or equal price than model B. So on the RHS of equation (A.19), we have $R1 < 1$. In addition, since $s_{A2} > s_{B2}$, then $R2 < 1$ in equation (A.19). Thus, the $RHS < 1$ for equation (A.19), which contradicts the result that $LHS > 1$.

Therefore, in the first period, the price of model A is higher than model B, $p^{*s}_{A1} > p^{*s}_{B1}$, when $n_{A2} > n_{B2}$ and the $n_{A1} > n_{B1}$. In the next step, we show that $n_{A2} > n_{B2}$ holds if $n_{A1} > n_{B1}$.

3. Step 3. This step shows that the manufacturer A keeps its OS network advantage to the second period: $n_{A2} > n_{B2}$ if $n_{A1} > n_{B1}$. Suppose on the contradictory that $n_{A2} \le n_{B2}$, then according to Step 1 result, $s_{B2} > s_{A2}$. So $LHS < 1$ for equation (A.19). Also $n_{A2} \le n_{B2}$ implies that $s_{A1} < s_{B1}$, so $R1 > 1$ for equation (A.19). In addition $p^{*s}_{A1} > p^{*s}_{B1}$ and that $s_{B2} > s_{A2}$ imply that $R2 > 1$ for equation (A.19). Hence, the $RHS > 1$, which contradicts that $LHS < 1$. Thus, $n_{A2} \le n_{B2}$ can't

hold, which means that $n_{A2} > n_{B2}$ holds by contradiction.

The three steps above shows that when the two manufacturers choose prices, the one with initial OS network advantage choose higher prices in both periods and keeps its advantage in the second period. So we have proved that: (1), $p^{*s}_{At} > p^{*s}_{Bt}$, for $t = 1, 2$; and (2), $n_{A2} > n_{B2}$.

$\square$